Technical Comments

Comment on "Nonadiabatic Temperature Distributions behind Detached Shocks"

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THE results obtained in the recent Note by Bluston[1] for higher orders in the radiation-convection parameter Γ are in error because the boundary conditions were applied incorrectly. At the shock wave, there is no incoming intensity $I^{(-)}$ if the freestream is cold; thus, in accordance with the differential approximation[2,3]

$$I - \beta q = 0 \qquad \text{at} \qquad y = 1 \tag{1}$$

where $\beta = (3)^{1/2}$ is perhaps best[3] but $\beta = 2$ is often used as well. At the black surface of the body, the outgoing intensity $I^{(+)}$ is

$$I + \beta q = T_w{}^4 \qquad \text{at} \qquad y = 0 \tag{2}$$

The author chose to add these two relations in order to obtain a single boundary condition on I, and this is obviously not correct. The regular perturbation

$$I = \sum_{i=0}^{\infty} B_i(t)\Gamma^i \qquad q = \sum_{i=0}^{\infty} C_i(t)\Gamma^i$$

leads to

$$B_i - \beta C_i = 0 \tag{3}$$

for all i at $t = 0$, and

$$B_0 + \beta C_0 = T_w{}^4 \tag{4a}$$

$$B_i + \beta C_i = 0 \qquad i \geq 1 \tag{4b}$$

as $t \rightarrow \infty$.

An alternate statement of the differential approximation leads to

$$dq/dt = -N\Gamma e^{-t}\{(v + 1)^4 - I\} \qquad dI/dt = 3N\Gamma e^{-t}q$$

Of course, Eq. (16) of Ref. 1 follows from the foregoing. The boundary conditions that Bluston used reproduce Eqs. (4a) and (3) to lowest order, but do not lead to Eq. (4b). The situation inherently depends upon the two-point boundary values for the half-range intensities $I^{(+)}$ and $I^{(-)}$, which are linear combinations of I and q (or dI/dt). The application of the correct boundary data yields a term of order Γ in I and modifies the temperature distribution as follows:

$$T = 1 + \Gamma(1 - B_0)\log y + \Gamma^2\{3NC_0 y - 2(B_0 - 1)\log^2 y - 3NC_0 + \gamma_1 \log y\} + 0(\Gamma^3) \tag{5}$$

where

$$B_0 = \tfrac{1}{2}T_w{}^4 \qquad C_0 = \tfrac{1}{2}(T_w{}^4/\beta)$$

$$\gamma_1 = \tfrac{1}{2}\{3NC_0 + N\beta(1 - B_0)\}$$

If $T_w = T_{w_1} + \Gamma T_{w_2} + \Gamma^2 T_{w_3} + \ldots$, where $T_{w_k} \sim 0(1)$, as in fact should be the case for a consistent regular perturbation, Eq. (4b) would be modified accordingly. In addition, the solution is invalid near the surface just as Goulard's[4] was.

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References

[1] Bluston, H. S., "Nonadiabatic Temperature Distributions behind Detached Shocks," *AIAA Journal*, Vol. 5, No. 11, Nov. 1967, pp. 2061–2062.

[2] Cheng, P., "Dynamics of a Radiating Gas with Application to Flow Over a Wavy Wall," *AIAA Journal*, Vol. 4, No. 2, Feb. 1966, pp. 238–245.

[3] Finkleman, D., "A Characteristics Approach to Radiation Gasdynamics," Paper 68-163, Jan. 1968, AIAA.

[4] Goulard, R., "The Coupling of Radiation and Convection in Detached Shock Layers," *Journal of Quantitative Spectroscopy & Radiative Transfer*, Vol. 1, No. 3/4, Dec. 1961, pp. 249–257.

Reply by Author to D. Finkleman

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THE author wishes to thank Mr. Finkleman for his interest in his paper. His comments[1] suggest, however, that he has not read the paper very carefully. Finkleman states that the solution is invalid near the surface. The last section of my paper[2] is explicitly concerned with the validity of the solution as a function of distance from the wall. It is clearly stated that at the wall the solution "converges for no physically realizable values of Γ" and "the range of Γ values for which the solution (8) is valid decreases . . . to 0.003 close to the body." Finkleman's comment here is, therefore, merely repeating a point that has already been made. As far as is known, the only solution valid near the wall is that of Melnik,[3] which is summarized in Ref. 4.

Turning to my application of the boundary conditions, Finkleman clearly has missed the point of the argument. The conditions are obtained at the instant before the postshock flow starts to radiate, and at this instant I, q = const over the entire shock layer. Under these conditions, it is quite legitimate to add the two relations (1) and (2) (of Ref. 1) to obtain my boundary conditions at $t = 0$. Finkleman's Eq. (4b) is obviously incorrect as his series expansions for I, q are invalid at the wall where $t \rightarrow \infty$ (Refs. 1 and 2). That is why it is necessary to apply both boundary conditions at $t = 0$ just before the postshock flow starts to radiate.

The author would like, however, to point out a misprint in line 15 of Ref. 2. This should read,

$$h \gg u^2 \text{ and } h \propto T$$

References

[1] Finkleman, D., "Comment on 'Nonadiabatic Temperature Distributions behind Detached Shocks,'" *AIAA Journal*, Vol. 6, No. 10, Oct. 1968, p. 2237.

[2] Bluston, H. S., "Nonadiabatic Temperature Distributions behind Detached Shocks," *AIAA Journal*, Vol. 5, No. 11, Nov. 1967, pp. 2061–2062.

[3] Melnik, R., private communication, Feb. 1966.

[4] Bluston, H. S., "The Application of Belotserkovskii's Method to the Nonadiabatic Flow around a Blunt Body," Rept. ADR 01-01-68.1, May 1968, Grumman Aircraft Engineering Corp.

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